A STATISTICAL TEST FOR 85TH AND 15TH PERCENTILE SPEEDS USING THE ASYMPTOTIC DISTRIBUTION OF SAMPLE QUANTILES

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Submission Date: August 1, 2011  
Word Count: 6896 (4146 text plus 2750 (3 figures and 8 tables))

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ABSTRACT

The 85th and the 15th percentiles are two parameters that are commonly used in traffic engineering and traffic safety. For example, 85th percentile speeds are used in setting speed limits and for evaluating the effectiveness of safety countermeasures, and 15th percentile speeds are used for establishing typical walking speeds for traffic signal timing. But the lack of a simple statistical test for percentiles led some researchers to omit statistical analysis or apply non-ideal approaches such as averaging 85th percentiles or using the binomial proportion test. This paper presents a statistical test for the 85th and 15th percentiles based on Crammer’s theory of asymptotic distribution of sample quantiles. This test is simpler to apply than the double bootstrapping and the quantile regression methods. To illustrate the practical use of this quantile test, it is applied to three sample applications: sequential warning lights at nighttime work zones, residential speed limits, and pedestrian signal timing.

Keywords: Quantiles, 15th percentile speed, 85th percentile speed, traffic safety
1. Introduction and Review of Literature on Percentiles

Percentiles, also known as quantiles, are often used in traffic engineering. For example, the 85th and 15th percentiles are commonly used in speed-related areas. This is especially true when safety is a main concern. The 85th percentile speed is the speed below which 85 percent of motorists travel and is the most frequently used metric for speed limit design (1). The MUTCD (2) guidelines on setting speed limits say, “When a speed limit is to be posted, it should be the 85th percentile speed of free-flowing traffic, rounded up to the nearest 10 km/h (5 mph) increment”. The TRB special report on managing speeds say that the 85th percentile speed is an important descriptive statistic in evaluating road safety (3). The assumption behind setting the speed limit is that the majority of drivers are rational, cautious and desire to select a maximum safe speed based on roadway conditions to minimize their travel time (4). In pedestrian speed studies, the 15th percentile speed represents the walking speed which can be exceeded by 85 percent of pedestrian population. The 15th percentile walking speed is used in pedestrian signal timing design, because a large proportion of the pedestrian population would be accommodated by adopting this value in design, and it appears to be a natural break point in the speed distribution curve (5).

The prevalence of percentiles in traffic engineering is the motivation for the development of a simple test statistic to assess the statistical significance of differences between two samples. The following section discusses the problems associated with existing approaches: complex procedure, non-ideal methodology, lack of details, and lack of statistical testing. Two existing sophisticated methods for comparing percentiles from two or more samples are the nonparametric double bootstrapping (6) and the quantile regression (7). Double nonparametric bootstrapping is a method of resampling existing data by using simulation. The first bootstrapping estimates the standard errors for desired percentiles. The second bootstrapping produces the confidence interval. This statistical test is beneficial in that it does not require populations to follow specific distributions and to have balanced sample sizes or equal variances. This method has been applied to a certain extent by others such as Brewer et al. (8) and Voigt et al. (9). Brewer et al. conducted a statistical test on the 85th percentile speed in a work zone speed limit study by applying the bootstrapping method. Voigt, et al. investigated the dual-advisory speed signing practice on freeway-to-freeway connectors in Texas and performed a statistical test on the 85th percentile speed by employing bootstrapping data analysis. Another method, the quantile regression, is a common econometric technique of extending the least squares estimate of the conditional means to the conditional quantiles. This method builds a linear model relating desired quantiles to intervention factors then estimates the standard error of desired quantiles through the standard error of model parameters. Both the double bootstrapping and the quantile regression methods are fairly complex methods that do not lend themselves to be easily applied en masse.

In some studies, the method applied was perhaps not the most appropriate fit such as averaging 85th percentiles or using the binomial test. Hildebrand et al. (10) analyzed 85th percentile speeds by averaging 85th percentile speeds from many work zone sites and then applying a t-test for the means. Agent et al. (11), performed statistical analysis on the average change in the 85th percentile speed at several data collection sites in their speed limit study in Kentucky. Mattox et al. (12) evaluated the effectiveness of speed-activated signs in work zones by performing statistical tests on the average reduction in 85th percentile speed from several speed data sets. The aforementioned three examples show that some have focused on the
sampling distribution of the mean in place of the sampling distribution of the percentile which one could argue is more appropriate. Some studies have applied the binomial test for analyzing percentiles such as the use of 85th percentile speeds for evaluating speed monitoring displays (13). The application of the binomial test for analyzing driver compliance percentages or proportions is appropriate, but its use for analyzing percentile values is questionable.

Other studies have not described their methodology in detail or have not pursued statistical analysis. For example, Knoblauch, et al. (5) conducted a statistical study of 15th percentile pedestrian speeds. In the study, a test statistic was used that was similar to the one presented in this paper, but the estimated value of the standard error was somewhat different. Because no details or references were provided in the study, the source of the formulation could not be traced. Another example involved the investigation of drone radars to reduce speeds in work zones (14). The article stated that a parametric hypothesis test could not be used to test statistical significance of the change in 85th percentile speed, and no statistical analysis was performed on the 85th percentile speeds.

The existing literature shows that there is a lack of a statistical test for percentiles that can be easily applied and is theoretically sound. The theory of the asymptotic distribution of sample quantiles has been in existence for many years as derived by Crammer (15). This paper presents a simple test statistic for comparing percentiles from two populations with independent normal distributions and illustrates the use of the test statistic with three case studies: 85th percentile vehicle speeds in work zones, 85th percentile vehicle speeds on residential streets and 15th percentile pedestrian speeds in central business districts.

2. Methodology

The statistical test for the 15th and 85th percentile speeds is based on the asymptotic distribution of sample quantiles. Because of the symmetry of the normal distribution, the test statistics for 15th and 85th percentile speeds have the same form. For conciseness, only the statistical test for 85th percentile speed is fully developed in this paper. First, Crammer’s (15) derivation of the general sample quantile distribution will be outlined. Then the standard error of the 85th sample quantile will be derived from the normal distribution. Last, the statistical test for the comparison of two 85th sample quantiles from independent normal distributions will be presented.

2.1. Asymptotic distribution of sample quantiles

Suppose \( X_1, X_2, \ldots, X_n \) are \( n \) independent and identically distributed random variables from a continuous one-dimensional distribution with cumulative distribution function \( F(x) \) and probability density function \( f(x) \). Let \( \zeta_p = F^{-1}(p) \) denote the quantile of the distribution of order \( p \) \((0 < p < 1)\). The random variables \( X_1, X_2, \ldots, X_n \) are arranged in ascending order of magnitude: \( X_{(1)} \leq X_{(2)} \leq X_{(3)} \ldots \leq X_{(n)} \), where random variable \( X_{(i)} \) is defined as the \( i^{th} \) smallest value in \( X_1, X_2, \ldots, X_n \). Then, \( X_{\lfloor np \rfloor + 1} \) is denoted as the quantile of the sample of order \( p \), where \( \lfloor np \rfloor \) is denoted as the greatest integer smaller or equal to \( np \).
Let \( g(x) \) denote the probability density function of random variable \( X_{(\lfloor np \rfloor + 1)} \). The marginal probability \( g(x)dx \) that \( X_{(\lfloor np \rfloor + 1)} \) is located in the interval \([x, x + dx]\) is equal to the probability that \( \lfloor np \rfloor \) sample values are smaller than \( x \), \( n - \lfloor np \rfloor - 1 \) sample values are greater than \( x + dx \), and 1 sample value is located in the interval between \( x \) and \( x + dx \) among \( n \) sample values. According to order statistics,

\[
g(x)dx = \left(\frac{n}{\lfloor np \rfloor}\right)(n - \lfloor np \rfloor)(F(x))^{\lfloor np \rfloor}(1 - F(x))^{n - \lfloor np \rfloor - 1} f(x)dx.
\]  

(1)

Using the DeMoivre-Laplace Central Limit Theorem, which states that a binomial distribution is approximately normal for large \( n \), as \( n \to \infty \),

\[
X_{(\lfloor np \rfloor + 1)} \sim N(\zeta_p, \frac{1}{f(\zeta_p)^2} \cdot \frac{p(1 - p)}{n}).
\]  

(2)

The previous derivation from Crammer of the sampling distribution of quantiles is equivalent to other derivations that apply the Delta method. The Delta method is considered a general central limit theorem that approximates a complex analytical variance computation by using a Taylor series (16).

2.2. Asymptotic distribution of 85th sample quantile from the normal distribution

In traffic studies, traffic speed distributions tend to have a normal distribution (17-21). The 85th percentile speed is of particular interest in traffic studies. The theory of asymptotic distribution of sample quantiles is then applied to the distribution of the 85th sample quantile from a normal distribution. Suppose \( X_1, X_2, \ldots, X_n \) are \( n \) independent and identically distributed random variables from a normal distribution with probability density function

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(1/2)[(x-\mu)/\sigma]^2}}.
\]  

(3)

The 85th sample quantile is denoted as \( X_{(\lfloor n0.85 \rfloor + 1)} \). The 85th distribution quantile is denoted as \( \zeta_{0.85} = 1.036\sigma + \mu \) with 1.036 being the 85th distribution quantile for standard normal. Then,

\[
f(\zeta_{0.85}) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(1/2)[(1.036\sigma + \mu - \mu)/\sigma]^2}} = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(1/2)(1.036)^2}} = 0.233/\sigma.
\]  

(4)

Based on the conclusion of asymptotic distribution of sample quantiles, for large \( n \),

\[
X_{(\lfloor n0.85 \rfloor + 1)} \sim N(\zeta_{0.85}, \frac{1}{f(\zeta_{0.85})^2} \cdot \frac{0.85(1 - 0.85)}{n})
\]  

(5)
By substituting (4) to (5),
\[
X_{(\lfloor n \cdot 0.85 \rfloor + 1)} \sim N(\zeta_{0.85}, 2.342\sigma^2 / n).
\] (6)

Thus, the random variable \( X_{(\lfloor n \cdot 0.85 \rfloor + 1)} - \zeta_{0.85} / (1.530\sigma / \sqrt{n}) \) is approximately standard normal for large \( n \). Monte Carlo simulations were performed to determine practical asymptotic sample sizes. The results indicated that when the sample size reached approximately 200, the sample distribution became asymptotically normal.

2.3. Comparing two 85th population quantiles from independent normal distributions

Let random variables \( X_{(\lfloor n \cdot 0.85 \rfloor + 1)} \) and \( Y_{(\lfloor n \cdot 0.85 \rfloor + 1)} \) be the 85th sample quantiles of independent random samples of size \( n_X \) and \( n_Y \) drawn from normal distributions with 85th distribution quantiles \( (\zeta_{0.85})_X \) and \( (\zeta_{0.85})_Y \) and variances \( \sigma_X \) and \( \sigma_Y \), respectively. Based on the derivations from the previous section, for large \( n_X \) and \( n_Y \), \( X_{(\lfloor n \cdot 0.85 \rfloor + 1)} \) and \( Y_{(\lfloor n \cdot 0.85 \rfloor + 1)} \) are approximately normally distributed with means \( (\zeta_{0.85})_X \) and \( (\zeta_{0.85})_Y \) and variances \( 1.530^2 \sigma_X^2 / n_X \) and \( 1.530^2 \sigma_Y^2 / n_Y \), respectively. Therefore, the random variable \( X_{(\lfloor n \cdot 0.85 \rfloor + 1)} - Y_{(\lfloor n \cdot 0.85 \rfloor + 1)} \) is also approximately normally distributed with mean \( (\zeta_{0.85})_X - (\zeta_{0.85})_Y \) and variance \( 1.530^2 \sigma_X^2 / n_X + 1.530^2 \sigma_Y^2 / n_Y \) (22). This implies that the random variable \( \frac{X_{(\lfloor n \cdot 0.85 \rfloor + 1)} - Y_{(\lfloor n \cdot 0.85 \rfloor + 1)}}{1.530\sqrt{\sigma_X^2 / n_X + \sigma_Y^2 / n_Y}} \) is standard normal. Since population variances, \( \sigma_X^2 \) and \( \sigma_Y^2 \), are often unknown, the sample variances \( S_X^2 \) and \( S_Y^2 \) are often substituted. For large sample sizes the sample variance approaches the population variance. Thus the random variable \( \frac{X_{(\lfloor n \cdot 0.85 \rfloor + 1)} - Y_{(\lfloor n \cdot 0.85 \rfloor + 1)}}{1.530\sqrt{S_X^2 / n_X + S_Y^2 / n_Y}} \) is still standard normal. In order to test the null hypothesis \( H_0 : (\zeta_{0.85})_X = (\zeta_{0.85})_Y \), which is equivalent to \( H_0 : (\zeta_{0.85})_X - (\zeta_{0.85})_Y = 0 \), the random variable \( \frac{X_{(\lfloor n \cdot 0.85 \rfloor + 1)} - Y_{(\lfloor n \cdot 0.85 \rfloor + 1)}}{1.530\sqrt{S_X^2 / n_X + S_Y^2 / n_Y}} \) can be used as the standard normal test statistic for examining the difference between two 85th population quantiles.

3. Case studies to illustrate the quantile test for 15th and 85th percentile speeds

The ease of use and the practical applicability of the quantile test for 15th and 85th percentile speeds are demonstrated using three case studies. Please note that the following case study summaries are only meant to illustrate the use of the quantile test and not to raise the substantive transportation engineering issues. All case studies involve large sample sizes which
meet the requirement of asymptotic distribution of sample quantiles. Normality of data was
evaluated using Q-Q (quantile-quantile) goodness of fit plots. The Q-Q plots indicated that the
speed distributions in all case studies were normal or very close to normal. The first case is an
analysis of the effectiveness of warning lights in nighttime work zone tapers. The second is an
investigation of the effects of reducing speed limits in residential areas. The third is a study of
pedestrian walking speed for signal timing design.

3.1. Effectiveness of warning lights in nighttime work zone tapers

Safety for drivers and construction workers is paramount in nighttime freeway work
zones. In the case of lane drops, drivers must reduce their speed and merge into the open lane
before barriers block their path of travel. One tool for alerting drivers of a nighttime work zone is
sequential lights. These lights flash in sequence and are attached to channelizers. In order to
investigate the effectiveness of sequential lights for improving safety, vehicles speeds were
measured with and without the deployment of such lights. The speed data was collected from
three short-term maintenance work zones on Interstate 70 in Missouri involving a right lane
closure (2 lanes dropped to 1 lane).

Descriptive statistics of speeds are presented in Table 1. The t-test was performed on the
speed data to determine if the mean speeds and the speed standard deviations were significantly
different after the installation of sequential lights. The results of t-test are shown in Table 2 and
indicate that the decrease in mean speeds was statistically significant. The mean does not directly
relate to the speed limit thus the 85% speed, should also be examined.

<table>
<thead>
<tr>
<th>TABLE 1 Speed statistics for case study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without lights</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Mean speed (mph)</td>
</tr>
<tr>
<td>85th percentile speed (mph)</td>
</tr>
<tr>
<td>Standard Deviation (mph)</td>
</tr>
<tr>
<td>Count (vehicles)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 2 T-test results for speeds for case study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothesis</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>( H_0: \mu_{\text{with}} = \mu_{\text{without}} )</td>
</tr>
<tr>
<td>( H_1: \mu_{\text{with}} &lt; \mu_{\text{without}} )</td>
</tr>
</tbody>
</table>

Key:
\( \mu_{\text{with}} \) is the mean speed of vehicles at work zones with sequential warning lights
\( \mu_{\text{without}} \) is the mean speed of vehicles at work zones without sequential warning lights

As shown in Figure 1, the cumulative speed distributions with and without sequential
lights have normal-like curves. The quantile test developed in this paper was applied to 85th
percentile speeds. Results of the test are shown in Table 3. The null hypothesis was rejected,
which suggested that the difference of 85th percentile speed was statistically significant.
Although only a 1 mph reduction in 85th percentile speeds resulted after sequential lights were
deployed, the reduction was statistically significant.
Figure 1 Cumulative speed distributions with and without sequential lights.

### Table 3 Result of statistical test on 85th percentile speed for case study 1

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>85% speed without lights (mph)</th>
<th>85% speed with lights (mph)</th>
<th>Change (mph)</th>
<th>P-Value</th>
<th>Reject null hypothesis?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: (\zeta_{0.85})<em>{\text{with}} = (\zeta</em>{0.85})_{\text{without}}$</td>
<td>63</td>
<td>62</td>
<td>-1</td>
<td>0.003</td>
<td>Yes</td>
</tr>
<tr>
<td>$H_1: (\zeta_{0.85})<em>{\text{with}} &lt; (\zeta</em>{0.85})_{\text{without}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key:
- $(\zeta_{0.85})_{\text{with}}$ is the 85th percentile speed with warning lights deployed at work zones
- $(\zeta_{0.85})_{\text{without}}$ is the 85th percentile speed without warning lights deployed at work zones

3.2. Residential speed limit reduction

Speeding on residential/neighborhood streets is a common citizen complaint to city councils. A study was conducted to investigate the effects of a posted speed limit reduction from 30 mph to 25 mph in Columbia, Missouri. Speed data was collected in two neighborhoods: Rothwell Heights and Shepard Boulevard. The study in Rothwell had two stages: no treatment and reduced speed limit by 5 mph (30 mph to 25 mph). The study in Shepard had the same stages as Rothwell with an additional education campaign after reducing the speed limit. The first stage consisted of speeds from two streets in each neighborhood before lowering the posted speed limits and after lowering speed limits. For the second stage, the new speed limit signs installed in the Shepard Boulevard neighborhood were oversized and had an attention-attracting yellow border. The last stage was to determine if an educational campaign would provide further reductions in average speeds, even if a speed reduction had already occurred during stage two of the methodology. Table 4 presents the summary of speed data from Rothwell Street in Rothwell Heights and Audubon Street in Shepard Boulevard.
The t-test was used to assess the statistical significance of the difference in mean speeds. The t-test results shown in Table 5 indicate that all of the streets experienced statistically significant reductions in mean speed.

### TABLE 5 T-test results for speeds for case study 2

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Mean speed before speed limit reduction or education (mph)</th>
<th>Mean speed after speed limit reduction or education (mph)</th>
<th>Change (mph)</th>
<th>P-value</th>
<th>Reject null hypothesis?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_{30}^R = \mu_{25}^R$; $H_1: \mu_{30}^R &gt; \mu_{25}^R$</td>
<td>37</td>
<td>31</td>
<td>-6</td>
<td>0.000</td>
<td>Yes</td>
</tr>
<tr>
<td>$H_0: \mu_{30}^A = \mu_{25}^A$; $H_1: \mu_{30}^A &gt; \mu_{25}^A$</td>
<td>29</td>
<td>28</td>
<td>-1</td>
<td>0.000</td>
<td>Yes</td>
</tr>
<tr>
<td>$H_0: \mu_{before}^A = \mu_{after}^A$; $H_1: \mu_{before}^A &gt; \mu_{after}^A$</td>
<td>28</td>
<td>27</td>
<td>-1</td>
<td>0.000</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Key:
- $\mu_{30}^R$ is the mean speed at speed limit 30 mph on Rothwell Street
- $\mu_{25}^R$ is the mean speed at speed limit 25 mph on Rothwell Street
- $\mu_{30}^A$ is the mean speed at speed limit 30 mph on Audubon Street
- $\mu_{25}^A$ is the mean speed at speed limit 25 mph on Audubon Street
- $\mu_{before}^A$ is the mean speed at speed limit 25 mph before education campaign on Audubon Street
- $\mu_{after}^A$ is the mean speed at speed limit 25 mph after education campaign on Audubon Street

Figures 2 a, b and c along with Q-Q plots demonstrate that the cumulative speed distributions of all data sets are normal-like. Three comparisons of 85th percentile speeds were made: (1) between 30 mph speed limit and 25 mph speed limit on Rothwell Street, (2) between 30 mph speed limit and 25 mph speed limit on Audubon Street, and (3) between before education campaign and after education campaign on Audubon Street. Results of the 85th percentile speed tests are shown in Table 6. Results indicate that the reduction of the speed limit and the education campaign both achieved a statistically significant decrease for the 85th percentile speed. In this case, it was difficult to tell if the reduction in 85th percentile speed after speed limit reduction and after the education campaign were statistically significant. In
comparisons (2) and (3), a less than 1 mph reduction occurred in 85\(^{th}\) percentile speed, but they were still found to be statistically significant.

(a) Cumulative speed distributions with speed limit 30 mph and 25 mph in Rothwell

(b) Cumulative speed distributions with speed limit 30 mph and 25 mph in Audubon
(c) Cumulative speed distributions before and after education in Audubon Street

FIGURE 2 Cumulative speed distributions.

TABLE 6 Result of quantile test on 85th percentile speed for case study 2.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>85% speed before speed limit reduction or education (mph)</th>
<th>85% speed after speed limit reduction or education (mph)</th>
<th>Change (mph)</th>
<th>P-value</th>
<th>Reject null hypothesis?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \left( \zeta_{0.85} \right)<em>{30}^R = \left( \zeta</em>{0.85} \right)_{25}^R$</td>
<td>45.00</td>
<td>38.00</td>
<td>-7.00</td>
<td>0.000</td>
<td>Yes</td>
</tr>
<tr>
<td>$H_1: \left( \zeta_{0.85} \right)<em>{30}^R &gt; \left( \zeta</em>{0.85} \right)_{25}^R$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0: \left( \zeta_{0.85} \right)<em>{30}^A = \left( \zeta</em>{0.85} \right)_{25}^A$</td>
<td>33.33</td>
<td>32.50</td>
<td>-0.83</td>
<td>0.001</td>
<td>Yes</td>
</tr>
<tr>
<td>$H_1: \left( \zeta_{0.85} \right)<em>{30}^A &gt; \left( \zeta</em>{0.85} \right)_{25}^A$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0: \left( \zeta_{0.85} \right)<em>{after}^A = \left( \zeta</em>{0.85} \right)_{before}^A$</td>
<td>32.50</td>
<td>32.00</td>
<td>-0.50</td>
<td>0.048</td>
<td>Yes</td>
</tr>
<tr>
<td>$H_1: \left( \zeta_{0.85} \right)<em>{after}^A &gt; \left( \zeta</em>{0.85} \right)_{before}^A$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key:
- $\left( \zeta_{0.85} \right)_{30}^R$ is the 85th percentile speed at speed limit 30 mph on Rothwell Street
- $\left( \zeta_{0.85} \right)_{25}^R$ is the 85th percentile speed at speed limit 25 mph on Rothwell Street
- $\left( \zeta_{0.85} \right)_{30}^A$ is the 85th percentile speed at speed limit 30 mph on Audubon Street
- $\left( \zeta_{0.85} \right)_{25}^A$ is the 85th percentile speed at speed limit 25 mph on Audubon Street
- $\left( \zeta_{0.85} \right)_{before}^A$ is the 85th percentile speed at speed limit 25 mph before education campaign on Audubon Street
- $\left( \zeta_{0.85} \right)_{after}^A$ is the 85th percentile speed at speed limit 25 mph after education campaign on Audubon Street

3.3. Signal timing for pedestrians

Pedestrian safety is of great concern in signal timing design at intersections. A balanced timing plan not only guarantees enough pedestrian crossing time but also allows for good traffic
flow through the intersection. Currently, the 15\textsuperscript{th} percentile speed of pedestrians is recommended for signal timing design. The 2009 edition of manual of uniform traffic control devices (MUTCD) recommends using a pedestrian walking speed of 3.5 ft/sec at intersections equipped with pedestrian signal heads. Field data was collected to measure the actual walking speed of pedestrians. Data was collected on the 9\textsuperscript{th} street in Columbia, Missouri, from 2:47 p.m. to 4:45 p.m. The descriptive statistics of pedestrian speeds is displayed in Table 7.

<table>
<thead>
<tr>
<th>Mean Speed (ft/s)</th>
<th>15th Percentile (ft/s)</th>
<th>Standard Deviation (ft/s)</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.48</td>
<td>3.76</td>
<td>0.72</td>
<td>235</td>
</tr>
</tbody>
</table>

Figure 3 shows the cumulative distribution of pedestrian speeds, which has a shape of normal distribution. The quantile test was used to make a statistical inference on the 15\textsuperscript{th} percentile speed. Based on the asymptotic normal approximation of sample quantiles, the test statistic is given by

\[
\frac{X_{(\lfloor n \cdot 0.15 \rfloor \cdot 1)} - (\zeta_{0.15})}{1.530 \sqrt{n}} \]

where \((\zeta_{0.15})\) is the hypothesized value of the 15\textsuperscript{th} population quantile or 3.5ft/sec in this case study. The test result is shown in Table 8. The null hypothesis was rejected at 95\% confidence level, meaning the pedestrian design speed for signal timing is statistically lower than the actual 15\textsuperscript{th} percentile speed of pedestrians.

![FIGURE 3 Cumulative speed distribution of pedestrian speeds](image)
Simple-to-use test statistics such as t-test, ANOVA and F-test are often used to analyze sample parameters. In the area of transportation safety, a critical parameter is the 15th and 85th percentile speeds and not the mean speed. The 15th and 85th percentile speed statistics are often described without statistical inference due to the lack of a simple statistical test. In this paper, the quantile test for comparing 15th and 85th sample percentiles is developed using Crammer’s derivation of the asymptotic distribution of sample quantiles. The simple quantile test provides the standard errors of the 15th and 85th sample quantiles. The quantile test is then illustrated through three case studies related to transportation safety: work zone safety, speed limit, and pedestrian walking speeds. These case studies illustrate that the quantile test is easy to apply and can be a useful tool to determine the statistical significance of changes due to treatments or countermeasures. The common assumption of a normally distributed sample is also applicable to the quantile test. The accuracy of the quantile test is compromised if the data is not normally distributed. But often, transportation data and specifically speed data is normally distributed or near normal. A related issue is that since the test is based on asymptotic properties, a large sample size is required. Monte Carlo simulations indicated that a sample size of greater than 200 is adequate for performing the quantile test for percentile speeds.

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